Designing a Stochastic Model to Analyze Online Course Delivery in e-Learning

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(Abstract) This paper concerns with a stochastic model to control an online course delivery in an e-learning system. Online course delivery is a general issue in e-learning systems. Integrating the activities is difficult due to rapid information flow and electronic devices failures. On the other hand, stochastic behaviour of course delivery activities make the situation worse. Here, we propose a program evaluation and review technique (PERT) to control the course delivery process. Also, for overcoming underestimation of the course delivery process completion time due to alternate paths, we apply Monte Carlo simulation. A numerical illustration shows the applicability of the proposed model.

Keywords: E-learning; Online course delivery; Program evaluation and review technique (PERT); Monte Carlo simulation

1. Introduction

Information and communication technologies promotion has provided new ideas in internet based environments and services. Teaching and learning through the internet enables the students to learn from anywhere and the instructor transform their teaching methods (Ozkan and Koseler, 2009; Czerniewicz and Brown, 2009). The new ideas about e-learning additional support are increasing, as learning through the internet is getting more popular. (Çobanoğlu et al., 2009).

In the field of e-learning optimization several works have been conducted in recent years. For comparing the e-learning system and traditional learning system, two comprehensive studies are illustrated. Mahdavi et al., (2008), compared traditional system with virtual educational system statistically in Iran. In this way, by the means of economical equations and statistical analysis they illustrated an in depth survey. Finally, by the means of hypothesis testing, they illustrated the best option for educational system is the combination of both systems. Fazlollahtabar and Sharma (2008), compared traditional engineering educational system with the e-learning engineering educational system on the economic dimension using hypothesis testing approach in Iran. The comparison involved trend analysis and prediction based on costs and benefits of the two systems. Interestingly, the analysis revealed that the traditional system had greater advantage on the economic dimension. Several factors support the e-learning system despite the associated economic disadvantage. The final analysis provided results in favor of a blended system which takes advantage of the traditional and e-learning systems.

Different studies have been worked out on cost optimization within e-learning environment. Mahdavi et al.,

(2008), identified varied cost elements in e-learning educational system and optimized them by the means of mathematical programming. Then they proposed an effective method to estimate the learning cost between any two skills of learner using the grey relational analysis. Mahdavi et al., (2008), developed their previous study combining the grey relational analysis and a radial basis function network to estimate the learning cost between any two skills after identification of varied cost elements in e-learning educational system and optimization by the means of mathematical programming. Fazlollahtabar and Yousefpoor (2009), applied the cost elements in the e-learning educational systems and proposed a combination of grey relational analysis and a radial basis function network to estimate the learning cost between any two skills. An integer programming method was employed to demonstrate that it is possible to facilitate the acquisition of single skills by considering a set of useful compound skills.

Finding the optimal (shortest) learning path for user or tutor has been studied in different works. Fazlollahtabar (2008), applied a dynamic programming to find the shortest path for users in the e-learning environment. Since the learning parameters are qualitative, he used an analytical hierarchy process approach (AHP) to turn the qualitative parameters into quantitative ones. Fazlollahtabar and Mahdavi (2009), proposed a neuro-fuzzy approach based on an evolutionary technique to obtain an optimal learning path for both instructor and learner. The neuro-fuzzy implementation helps to encode both structured and non-structured knowledge for the instructor. On the other hand, for learners, the neural network approach has been applied to make personalized curriculum profile based on individual learner requirements in a fuzzy environment.

Also Tajdin et al., (2008), designed an assessment method based on real-time simulators. These simulators were able to facilitate education and play the role of virtual intelligent teacher referring to student capabilities by following the feedback mechanisms. This system, which was constructed by the means of network and expert system, was contained a real-time simulator core that has an inference engine based on a hypothesis testing.

For analyzing user satisfaction in e-learning system, Mahdavi et al., (2008), designed a heuristic methodology for multi-criteria evaluation of web-based e-learning systems based on the theory of multi-criteria decision making and the research results concerning user satisfaction in the fields of human-computer interaction and information systems.

Online course delivery is a set of activities sequenced to transmit learning content to students by including computers and the internet in the learning process. The principal components of an online course delivery package include curriculum mapping (breaking curriculum into sections that can be assigned and assessed), student tracking, online support for both teacher and student, electronic communication (e-mail, threaded discussions, chat, Web publishing), and internet links to outside curriculum resources. In general, e-learning users are assigned either a teacher ID or a student ID (Krause, et al., 2009; Lykourentzou, et al., 2009).

In this paper, we propose a new idea for useful e-learning support using PERT project planning method. According to some academic standards knowledge must be "measured" in quantity as well as in time. That kind of valuation is more difficult in case of virtual learning. E-learning process consists of a group of activities that a user should perform to gain the learning content and objectives. The applications of electronic facilities within virtual learning system are of important due to time that they incur. The nature of virtual environment implies uncertainty. Here, we propose a method to control the time uncertainty during the learning process in an e-learning system.

2. Online Course Delivery

Corporate IT training occurs by one of several scenarios. We identified that there are a few primary items, some or all of which are involved in any given system:

- Training provider
- Primary site contact
- Classroom coordinator
- Instructor
- Student

The resources are primarily organized based on the roles that they support, to make a delivery process as straightforward as possible. But each situation is different from others, therefore the resources should be designed in a way to be appropriate for multiple users. Then, for each course, there is a Delivery Guide, which is primarily intended for the training provider. This item explains and links to the other resources, so it's the best entry point to the rest of the process. The Classroom Setup Guide is primarily intended for a classroom coordinator — someone who is responsible for configuring a training room prior to class time. In some cases students are responsible for setting up their own laptops and bringing them to class. For this purpose the Setup Guide is suitable for distribution to students

The Setup Guide links various resources including supporting tools and lab software. Most tools for courses are bundled into a lab image, to minimize setup effort. Separate, interactive installations are usually only necessary for one or two tools, such as the Java Developer's Kit or EE SDK, or an IDE such as Eclipse. Most of these are free and downloadable; a few courses use commercial tools by design, and this will be flagged in the Setup Guide and also the course outline, under System Requirements.

The Instructor's Guide is designed just for the instructor. It is recommended that this item and the Setup Guide be provided to the instructor, along with a copy of the course book itself. This gives the instructor time to prepare. Even very experienced instructors who are experts on the subject under study will want to familiarize themselves with the course materials themselves. It is one thing to know the technology; it is another to know, for instance, how to run the first demonstration on the first day, or how a particular lab exercise progresses, so as to be able to answer students' questions. A configuration of a proposed online course delivery is shown in Figure 1.
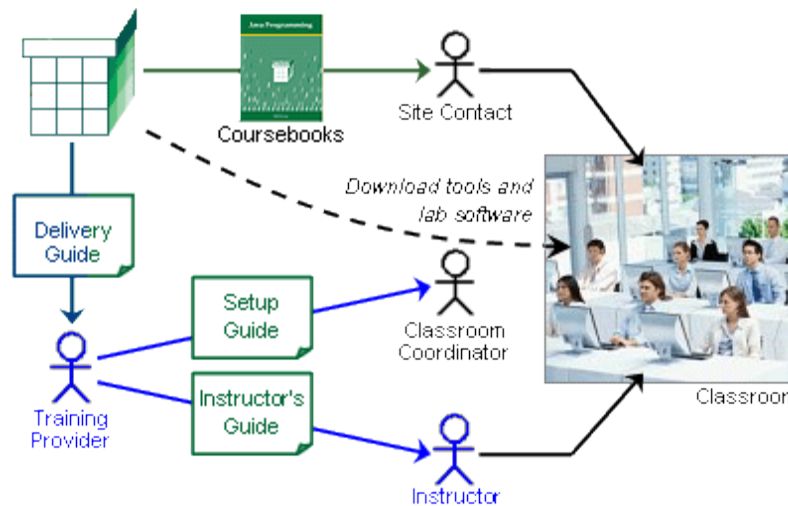
Figure 1. A configuration of a proposed online course delivery

Now, we discuss on a delivery system for the online course delivery, commonly referred to as a Learning Management System, or LMS. Some factors should be noticed proposing an LMS,

1. The origin of the delivery system
2. The feature of an LMS
3. The capability of facilitating learning process
4. The access amount for a proposed LMS
5. An appropriate administrator for LMS

Providing the LMS from a vendor or by internal expertise, namely the IT department considers benefits and drawbacks for both.

A vendor-purchased system is typically made-to-order by a group that specializes in just that. One can give the vendor your organization's specifications and they can come back with a system design and price. On the drawback side, the specifications can sometimes be costly - and changes can be slow and costly, as well.

A homegrown LMS is fantastic if you have the expertise and resources in-house to build it. An in-house development team knows the organization, its growth, and its audience. Plus, a homegrown system may cost less in the long run. The potential roadblocks to this path are that the expertise must be present in-house - and they must be able to devote themselves pretty much full time to the project.

Once the decision to go internal or external for the build-out of the delivery system has been made, thinking about the features begins. Some facilities such as tracking completion and recording grades, registering and tracking traditional learning programs, enabling learners to print a certificate of completion for their online courses.

On the technical side, the LMS need to have the capability of creating complex video streaming, interactive modules, and collaboration. Therefore, the status of existing technology and its accommodation to the LMS should be noticed.

The population who is going to access the delivery system is another important factor. Can the organization's infrastructure and bandwidth handle the influx of users? What about access? Enabling users to access the system from the internet, or within the confines of the organization's firewall, is a crucial enquiry.

Typically a person (or more than one person) is needed to administer the system, even if it is purchased from a vendor. The decisions about the structure of the technology may also affect the staffing.

3. Controlling Learning Process

Course delivery process requires a series of activities, some of which must be performed sequentially and others that can be performed in parallel with other activities. This collection of series and parallel tasks can be modeled as a network. In 1957 the Critical Path Method (CPM) was developed as a network model for project management. CPM is a deterministic method that uses a fixed time estimate for each activity. While CPM is easy to understand and use, it does not consider the time variations that can have a great impact on the completion time of a course delivery process. The Program Evaluation and Review Technique (PERT) is a network model that allows for randomness in activity completion times. It has the potential to reduce both the time and cost required to complete a course delivery process. In our proposed model, we consider the sequence of course delivery activities. We apply PERT to estimate the uncertain time of the online course delivery activities in e-learning.

3.1. The Network Diagram

In a course delivery process, an activity is a task that must be performed and an event is a milestone marking the completion of one or more activities. Before an activity can begin, all of its predecessor activities must be completed. Learning process network models represent activities and milestones by arcs and nodes. PERT originally was an activity on arc network, in which the activities are represented on the lines and milestones on the nodes. Over time, some people began to use PERT as an activity on node network. The PERT chart may have multiple pages with many sub-tasks. Figure 1
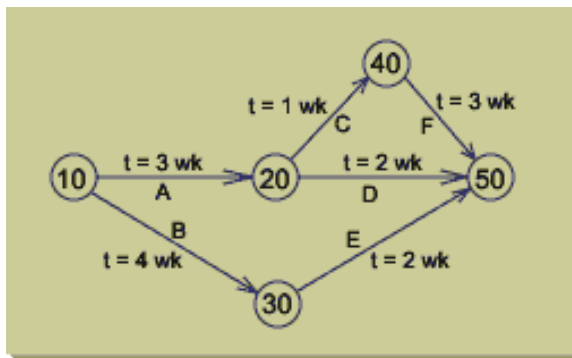
is a very simple example of a PERT diagram.

Figure 2. A simple example of a PERT diagram

The milestones generally are numbered so that the ending node of an activity has a higher number than the beginning node. Incrementing the numbers by 10 allows for new ones to be inserted without modifying the numbering of the entire diagram. The activities in the above diagram are labeled with letters along with the expected time required to complete the activity.

PERT planning involves the following steps:

- Identify Activities and Milestones

The activities are the tasks required to complete the course delivery process. The milestones are the events marking the beginning and end of one or more activities. It is helpful to list the tasks in a table that in later steps can be expanded to include information on sequence and duration.

- Determine Activity Sequence

This step may be combined with the activity identification step since the activity sequence is evident for some tasks. Other tasks may require more analysis to determine the exact order in which they must be performed.

- Construct the Network Diagram

Using the activity sequence information, a network diagram can be drawn showing the sequence of the serial and parallel activities. For the original activity-on-arc model, the activities are depicted by arrowed lines and milestones are depicted by circles or "bubbles". If done manually, several drafts may be required to correctly portray the relationships among activities. Software packages simplify this step by automatically converting tabular activity information into a network diagram.

- Estimate Activity Times

Hours are a commonly used unit of time for activity completion, but any consistent unit of time can be used. A distinguishing feature of PERT is its ability to deal with uncertainty in activity completion times. For each activity, the model usually includes three time estimates:

Optimistic time - generally the shortest time in which the activity can be completed. It is common practice to specify optimistic times to be three standard deviations from the mean so that there is approximately a 1% chance that the activity will be completed within the optimistic time.

Most likely time - the completion time having the highest probability. Note that this time is different from the expected time.

Pessimistic time - the longest time that an activity might require. Three standard deviations from the mean is commonly used for the pessimistic time.

PERT assumes a beta probability distribution for the time estimates. For a beta distribution, the expected time for each activity can be approximated using the following weighted average:

Expected time = (Optimistic + 4 * Most likely + Pessimistic) / 6 (1)

This expected time may be displayed on the network diagram. To calculate the variance for each activity completion time, if three standard deviation times were selected for the optimistic and pessimistic times, then there are six standard deviations between them, so the variance is given by:

Variance = [(Pessimistic - Optimistic) / 6]2 (2)

- Determine the Critical Path

The critical path is determined by adding the times for the activities in each sequence and determining the longest path in the course delivery process. The critical path determines the total calendar time required for the course delivery process. If activities outside the critical path speed up or slow down (within limits), the total course delivery process time does not change. The amount of time that a non-critical path activity can be delayed without delaying the course delivery process is referred to as *slack time*. If the critical path is not immediately obvious, it may be helpful to determine the following four quantities for each activity:

ES - Earliest Start time

EF - Earliest Finish time

LS - Latest Start time

LF - Latest Finish time

These times are calculated using the expected time for the relevant activities. The earliest start and finish times of each activity are determined by working forward through the network and determining the earliest time at which an activity can start and finish considering its predecessor activities. The latest start and finish times are the latest times that an activity can start and finish without delaying the learning process. LS and LF are found by working backward through the network. The difference in the latest and earliest finish of each activity is that activity's slack. The critical path then is the path through the network in which none of the activities have slack. The variance in the course delivery process completion time can be calculated by summing the variances in the completion times of the activities in the critical path. Given this variance, one can calculate the probability that the learning process will be completed by a certain date assuming a normal probability distribution for the critical path. The normal distribution assumption holds if the number of activities in the path is large enough for the central limit theorem to be applied. Since the critical path determines the completion date of the course delivery process, the course delivery process can be accelerated by adding the resources required to decrease the time for the

activities in the critical path. Such a shortening of the learning process sometimes is referred to as *project crashing*.

- Update as Project Progresses

Make adjustments in the PERT chart as the project progresses. As the course delivery process unfolds, the estimated times can be replaced with actual times. In cases where there are delays, additional resources may be needed to stay on schedule and the PERT chart may be modified to reflect the new situation.

3.2. Benefits and Limitations of PERT

The benefits and limitations of the PERT are listed below:
Benefits:
- Expected project completion time
- Probability of completion before a specified date
- The critical path activities that directly impact the completion time
- The activities that have slack time and that can lend resources to critical path activities
- Activities start and end dates

Limitations:
The activity time estimates are somewhat subjective and depend on judgment. In cases where there is little experience in performing an activity, the numbers may be only a guess. In other cases, if the person or group performing the activity estimates the time there may be bias in the estimate.
Even if the activity times are well-estimated, PERT assumes a beta distribution for these time estimates, but the actual distribution may be different.

Even if the beta distribution assumption holds, PERT assumes that the probability distribution of the learning process completion time is the same as that of the critical path. Because other paths can become the critical path if their associated activities are delayed, PERT consistently underestimates the expected course delivery process completion time.

The underestimation of the course delivery process completion time due to alternate paths becoming critical is perhaps the most serious of these issues. To overcome this limitation, Monte Carlo simulations can be performed on the network to eliminate this optimistic bias in the expected learning process completion time.

4. Monte Carlo simulation

Monte Carlo simulation is a comprehensive approach for analyzing the behavior of some activities, plans or processes that involve uncertainty. If we face uncertain or variable market demand, fluctuating costs, variation in a manufacturing process, or effects of weather on operations, or stochastic activity time we can benefit from using Monte Carlo simulation to understand the impact of uncertainty, and develop plans to mitigate or otherwise cope with risk. Whenever we need to make an estimate, forecast or decision where there is significant uncertainty, we'd be well advised

to consider Monte Carlo simulation (Metropolis and Ulam, 1949).

Monte Carlo simulation is a method for iteratively evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than just a couple uncertain parameters. The Monte Carlo method is just one of many methods for analyzing uncertainty propagation, where the goal is to determine how random variation, lack of knowledge, or error affects the sensitivity, performance, or reliability of the system that is being modeled. Monte Carlo simulation is categorized as a sampling method because the inputs are randomly generated from probability distributions to simulate the process of sampling from an actual population. So, we try to choose a distribution for the inputs that most closely matches data we already have, or best represents our current state of knowledge. The data generated from the simulation can be represented as probability distributions (or histograms) or converted to error bars, reliability predictions, tolerance zones, and confidence intervals. (see Figure 3).
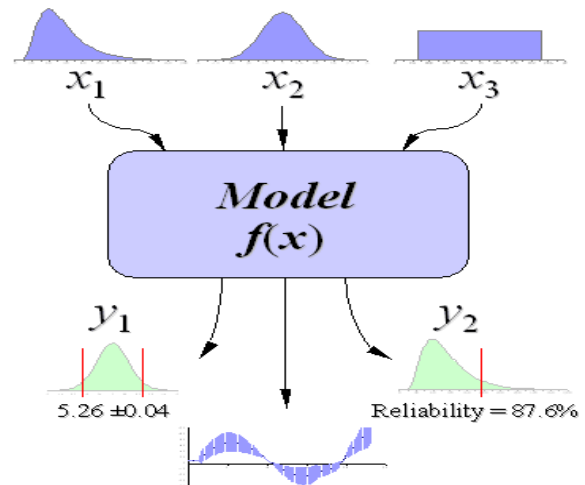

Figure 3. The basic principle behind Monte Carlo simulation

The steps in Monte Carlo simulation corresponding to the uncertainty shown in Figure 2 are generally simple, and can be easily implemented. The following five steps are proposed to implement Monte Carlo,

Step 1: Create a parametric model, $y = f(x_1, x_2, ..., x_q)$.
Step 2: Generate a set of random inputs, $x_{i1}, x_{i2}, ..., x_{iq}$.
Step 3: Evaluate the model and store the results as y_i.
Step 4: Repeat steps 2 and 3 for $i = 1$ to n.
Step 5: Analyze the results using histograms, summary statistics, confidence intervals, etc.

5. Numerical Example

Here, we propose an example to show the applicability and the effectiveness of our proposed model. The activities and their corresponding numbers are shown in Table 1.

Table 1. The activities and their corresponding number

Activity number	Activity
1	Connect
2	Present/absent
3	e-Library
4	Group work
5	e-Lab
6	Exam
7	Deliver project
8	Learning content study

Considering the relations between the activities, they configure a network diagram. The proposed network diagram is presented in Figure 4.
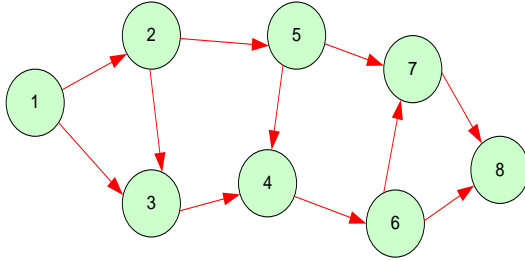

Figure 4.The proposed network diagram

To consider PERT for computing the completion time of an online course delivery, the optimistic, most likely, and pessimistic times are given in Table 2. Using these times and relations (1) and (2), we can calculate expected time, standard deviation and variance. The reports are given in Table 2.

Table 2. The results of the computations

Activity	t_o	t_m	t_p	t_e	S	S^2
1-2	1	4	7	4	1	1
1-3	8	10	18	11	1.66	2.77
2-3	1	2	3	2	0.33	0.11
3-4	1	3	11	4	1.66	2.77
2-5	1	2	3	2	0.33	0.11
4-5	2	3	4	3	0.33	0.11
4-6	7	8	9	8	0.33	0.11
5-7	2	3	4	3	0.33	0.11
6-7	1	2	3	2	0.33	0.11
6-8	1	4	7	4	1	1
7-8	3	7	11	7	1.33	1.77

As a result, the critical path is gained as follows:

Critical path =1-3-4-6-7-8

The expected time for the critical path is,

$$T_e = t_m\,(1\text{-}3) + t_m\,(3\text{-}4) + t_m\,(4\text{-}6) + t_m\,(6\text{-}7) + t_m\,(7\text{-}8)$$
=10+3+8+2+7=30

The total variance of the critical path is,

$$V_T = S_T^2 = S^2\,(1\text{-}3) + S^2\,(3\text{-}4) + S^2\,(4\text{-}6) + S^2\,(6\text{-}7) + S^2\,(7\text{-}8)$$
=2.77+2.77+0.11+0.11+1.77= 7.53
The total standard deviation is,
$$S_T = \sqrt{S^2} = \sqrt{7.53} = 2.74$$

To gain a 95% confidence for finishing the learning process at the appropriate time (T_P) we apply Equation (3),

$$Z = \frac{T_e - T_p}{S}$$

(3)

Therefore, we have

$$T_e = 30;\ S_T = 2.74;\ P\{T \le T_P\} = 0.95;$$
$$T_P = S * Z + T_e$$

T_P =2.74*1.645+30=34.5073

So, if the time is 34.5073 unit of time then we can express with 95% confidence that the project will be finished on time. Here, to verify our proposed PERT computations and also to overcome the drawbacks of PERT as stated in section 3.2, we apply Monte Carlo simulation approach. According to a

sampling procedure of activity times, we compute the variable times corresponding the cumulative probability of occurrence for each activity which is reported in Table 3.

Table 3. The variable times corresponding the probability of occurrence

Activity 1-2		Activity 1-3		Activity 2-3		Activity 2-5	
Cumulative Probability	Sorted times	Cumulative Probability	Sorted times	Cumulative Probability	Sorted times	Cumulative Probability	Sorted times
0.20	3.8	0.15	9.6	0.08	1.65	0.12	1.72
0.45	3.9	0.28	9.7	0.026	1.85	0.31	1.87
0.75	4	0.53	9.9	0.48	2	0.55	1.95
0.85	4.1	0.73	10	0.72	2.1	0.75	2
0.93	4.2	0.9	10.3	0.88	2.2	0.88	2.08
1	4.25	1	10.35	1	2.25	1	2.15

Activity 3-4		Activity 4-5		Activity 4-6		Activity 5-7	
Cumulative Probability	Sorted times	Cumulative Probability	Sorted times	Cumulative Probability	Sorted times	Cumulative Probability	Sorted times
0.13	2.8	0.07	2.76	0.20	7.79	0.1	2.81
0.33	2.9	0.16	2.84	0.42	7.94	0.27	2.9
0.58	2.95	0.4	2.94	0.65	8	0.49	2.95
0.82	3.05	0.6	3	0.83	8.02	0.72	3
0.94	3.12	0.85	3.08	0.93	8.1	0.9	3.06
1	3.2	1	3.15	1	8.14	1	3.13

Activity 6-7		Activity 6-8		Activity 7-8	
Cumulative Probability	Sorted times	Cumulative Probability	Sorted times	Cumulative Probability	Sorted times
0.1	1.84	0.1	3.8	0.13	6.71
0.25	1.96	0.25	3.9	0.33	6.85
0.51	2	0.49	3.96	0.56	6.95
0.77	2.04	0.74	4	0.79	7
0.81	2.1	0.9	4.05	0.94	7.05
1	2.15	1	4.1	1	7.1

Here, according to the Monte Carlo process we generate five series of random numbers as given in Table 4.

Table 4. The generated random numbers

	1-2	1-3	2-3	3-4	2-5	6-7	4-5	5-7	4-6	6-8	7-8
1	0.632175	0.642	0.381758	0.644788	0.338479	0.884028	0.075288	0.093674	0.831389	0.696565	0.852587
2	0.238231	0.0211	0.446354	0.584146	0.157217	0.467517	0.896606	0.206451	0.930826	0.419563	0.666832
3	0.567427	0.1123	0.278362	0.707245	0.927085	0.097144	0.996739	0.477217	0.039175	0.297241	0.084283
4	0.613292	.78532	0.872533	0.531375	0.666586	0.339968	0.861182	0.797166	0.04846	0.368277	0.195424
5	0.013479	0.34578	0.253833	0.436249	0.344377	0.735796	0.572929	0.397602	0.717149	0.177423	0.816713

Now, we accommodate the random numbers of each activity with the corresponding activity times in Table 3. The following activity times, Table 5, are obtained for the activities associated with the average time for each activity.

Table 5. The obtained activity times

	1-2	1-3	2-3	3-4	2-5	6-7	4-5	5-7	4-6	6-8	7-8
1	4	10	2	3.05	1.95	2.15	2.76	2.81	8.02	4	7.05
2	3.9	9.6	2	2.95	1.87	2	3.15	2.9	8.1	3.96	7
3	4	9.6	2	3.05	2.15	1.84	3.15	2.95	7.79	3.96	6.71
4	4	10.3	2.2	2.95	2	2	3.15	3.06	7.79	3.96	6.85
5	3.8	9.9	1.85	2.95	1.95	2.04	3	2.9	8.02	3.9	7.05
Average	3.94	9.88	2.01	2.99	1.984	2.006	3.042	2.924	7.944	3.956	6.932

Considering the critical path obtained in PERT computations, we have the following completion time.

$$T=9.88+2.99+7.944+2.006+6.932=29.752$$

Obviously, the total completion time in Monte Carlo approach is lower than the PERT. It implies that the Monte Carlo is more effective for considering stochastic times in total completion time computations.

6. Discussions

To evaluate the results obtained from the PERT and Monte Carlo simulation in online course delivery, some circumstances are considered and the related comparisons are set. First, let's consider the deterministic activity times, where we have to use, for instance, the expected value of the activities as a deterministic number in out computations. Doing that, the total time required for online course delivery obtained to be 37.43, which is worse than the two stochastic approaches. This is due to the inappropriate estimation (using expected value) of the course delivery activities. So, considering the significant aspects of stochastic parameter is effective on the total course delivery time.

Also, any stochastic parameter is presented using some past data. The more the data, the better is the estimation. Thus, Monte Carlo using more data for estimation is more appropriate than PERT using just few factors for estimation. Second, the result obtained in this study is compared to the ones reviewed in the literature. To do that the, the methodology used, the type of activity time, and the final results are summarized in Table 6.

Table 6. The comparison with other approaches

Methodology	Type of activity time	Final result
Fazlollahtabar, (2008)	Expected value	31.35
Fazlollahtabar, and Mahdavi, (2009)	Mapping membership functions	39.18
Fazlollahtabar, and Sharma, (2008)	Confidence interval using mean and variance for hypothesis testing	41.82
Fazlollahtabar, and Yousefpoor, (2009)	Expected value	36.44
PERT	Parameters explained in the paper	34.5073
Monte Carlo Simulation	Cumulative probability distribution	29.752

The results from different methodologies emphasize the effective ness of the Monte Carlo simulation.

7. Conclusions

Here, we proposed an approach to estimate the learning process time in an e-learning environment. Considering the stochastic and uncertain nature of online course delivery activities in virtual learning systems, PERT is applied for finding the expected time of the course delivery. To overcome the limitations of PERT, a Monte Carlo simulation is proposed. The effectiveness and applicability of the proposed model is illustrated in an example. We have shown that the model is capable to compute the total online course delivery process time with a confidence percentage. Also, Monte Carlo simulation implies better performance in computing total completion time of online course delivery process.

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